CHAPTER

6

Stress Concentration

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Mathematical analysis and experimental measurement show that in a loaded structural member, near changes in the section, distributions of stress occur in which the peak stress reaches much larger magnitudes than does the average stress over the section. This increase in peak stress near holes, grooves, notches, sharp corners, cracks, and other changes in section is called *stress concentration*. The section variation that causes the stress concentration is referred to as a *stress raiser*. Although an extensive collection of stress concentration factors is tabulated in this chapter, a much larger collection is provided in Ref. [6.1].

6.1 NOTATION

The units for some of the definitions are given in parentheses, using L for length and F for force.

K_ε	Effective strain concentration factor
K_f	Effective stress concentration factor for cyclic loading, fatigue notch factor
K_i	Effective stress concentration factor for impact loads
K_σ	Effective stress concentration factor
K_t	Theoretical stress concentration factor in elastic range, $= \sigma_{\max}/\sigma_{\text{nom}}$
q	Notch sensitivity index
q_f	Notch sensitivity index for cyclic loading
q_i	Notch sensitivity index for impact loading
r	Notch radius (L)

ε_{nom} Nominal strain (L/L)

σ_{nom} Nominal stress (F/L^2) of notched member; for example, for an extension member, σ_{nom} is usually taken to be the axial load divided by the cross-sectional area measured at the notch (i.e., area taken remotely from notch minus area corresponding to notch). In practice, the definition of the reference stress σ_{nom} depends on the problem at hand. In Table 6-1 the reference stress is defined for each particular stress concentration factor.

6.2 STRESS CONCENTRATION FACTORS

Figure 6-1 shows a large plate that contains a small circular hole. For an applied uniaxial tension the stress field is found from linear elasticity theory [6.2]. In polar coordinates the azimuthal component of stress at point P is given as

$$\sigma_\theta = \tfrac{1}{2}\sigma\left[1 + (r^2/\rho^2)\right] - \tfrac{1}{2}\sigma\left[1 + 3(r^4/\rho^4)\right]\cos 2\theta \tag{6.1}$$

The maximum stress occurs at the sides of the hole where $\rho = r$ and $\theta = \frac{1}{2}\pi$ or $\theta = \frac{3}{2}\pi$. At the hole sides,

$$\sigma_\theta = 3\sigma$$

The peak stress is three times the uniform stress σ.

To account for the peak in stress near a stress raiser, the *stress concentration factor* or *theoretical stress concentration factor* is defined as the ratio of the calculated peak stress to the nominal stress that would exist in the member if the distribution of stress

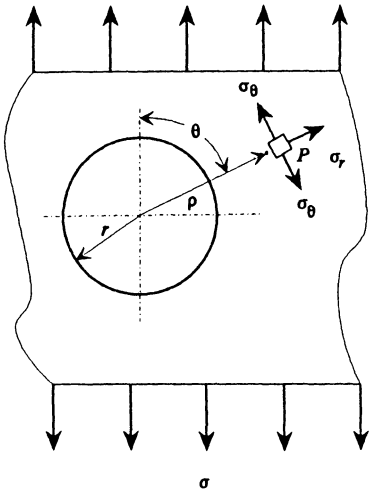


Figure 6-1: Infinite plate with a small circular hole.

remained uniform; that is,

$$K_t = \frac{\sigma_{\text{max}}}{\sigma_{\text{nom}}} \tag{6.2}$$

The nominal stress is found using basic strength-of-materials formulas, and the calculations can be based on the properties of the net cross section at the stress raiser. Sometimes the overall section is used in computing the nominal stress.

If σ is chosen as the nominal stress for the case shown in Fig. 6-1, the stress concentration factor is

$$K_t = \sigma_{\text{max}}/\sigma_{\text{nom}} = 3$$

The effect of the stress raiser is to change only the distribution of stress. Equilibrium requirements dictate that the average stress on the section be the same in the case of stress concentration as it would be if there were a uniform stress distribution. Stress concentration results not only in unusually high stresses near the stress raiser but also in unusually low stresses in the remainder of the section.

When more than one load acts on a notched member (e.g., combined tension, torsion, and bending) the nominal stress due to each load is multiplied by the stress concentration factor corresponding to each load, and the resultant stresses are found by superposition. However, when bending and axial loads act simultaneously, superposition can be applied only when bending moments due to the interaction of axial force and bending deflections are negligible compared to bending moments due to applied loads.

The stress concentration factors for a variety of member configurations and load types are shown in Table 6-1. A general discussion of stress concentration factors and factor values for many special cases are contained in the literature (e.g., [6.1]).

Example 6.1 Circular Shaft with a Groove The circular shaft shown in Fig. 6-2 is girdled by a U-shaped groove, with $h = 10.5$ mm deep. The radius of the groove root $r = 7$ mm, and the bar diameter away from the notch $D = 70$ mm. A bend-

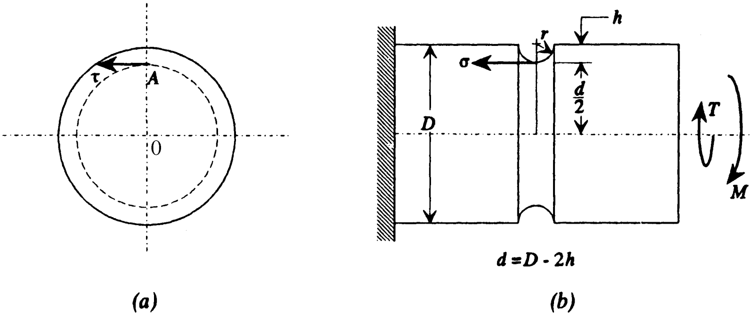


Figure 6-2: Circular shaft with a U-groove.

ing moment of 1.0 kN·m and a twisting moment of 2.5 kN·m act on the bar. The maximum shear stress at the root of the notch is to be calculated.

The stress concentration factor for bending is found from part I in Table 6-1, case 7b. Substitute

$$2h/D = \tfrac{21}{70} = 0.3, \qquad h/r = 10.5/7 = 1.5 \tag{1}$$

into the expression given for K_t:

$$K_t = C_1 + C_2(2h/D) + C_3(2h/D)^2 + C_4(2h/D)^3 \tag{2}$$

Since $0.25 \leq h/r = 1.5 < 2.0$, we find, for elastic bending,

$$C_1 = 0.594 + 2.958\sqrt{h/r} - 0.520h/r$$

with C_2, C_3, and C_4 given by analogous formulas in case I-7b of Table 6-1. These constants are computed as

$$C_1 = 3.44, \qquad C_2 = -8.45, \qquad C_3 = 11.38, \qquad C_4 = -5.40$$

It follows that for elastic bending

$$K_t = 3.44 - 8.45(0.3) + 11.38(0.3)^2 - 5.40(0.3)^3 = 1.78 \tag{3}$$

The tensile bending stress σ_{nom} is obtained from Eq. (3.56a) as $Md/2I$ and at the notch root the stress is

$$\sigma = K_t\frac{Md}{2I} = \frac{(1.78)(1.0 \times 10^3 \text{ N-m})(0.049 \text{ m})(64)}{2\pi(0.049)^4 \text{ m}^4} = 154.1 \text{ MPa} \tag{4}$$

The formulas from Table 6-1, part I, case 7c, for the elastic torsional load give $K_t = 1.41$. The nominal twisting stress at the base of the groove is [Eq. (3.48)]

$$\tau = \frac{K_t Td/2}{J} = \frac{K_t Td(32)}{2\pi d^4} = \frac{(1.41)(2.5 \times 10^3 \text{ N}\cdot\text{m})16}{\pi(0.049)^3} = 152.6 \text{ MPa} \tag{5}$$

The maximum shear stress at the base of the groove is one-half the difference of the maximum and minimum principal stresses (Chapter 3). The maximum principal stress is

$$\sigma_{\max} = \tfrac{1}{2}\sigma + \tfrac{1}{2}\sqrt{\sigma^2 + 4\tau^2} = \tfrac{1}{2}(154.1) + \tfrac{1}{2}\sqrt{154.1^2 + 4(152.6)^2} = 248.0 \text{ MPa}$$

and the minimum principal stress is

$$\sigma_{\min} = \tfrac{1}{2}\sigma - \tfrac{1}{2}\sqrt{\sigma^2 + 4\tau^2} = \tfrac{1}{2}(154.1) - \tfrac{1}{2}\sqrt{154.1^2 + 4(152.6)^2} = -93.9 \text{ MPa}$$

Thus, the maximum shear stress is

$$\tau_{\max} = \tfrac{1}{2}(\sigma_{\max} - \sigma_{\min}) = \tfrac{1}{2}(248.0 + 93.9) = 171.0\ \text{MPa} \tag{6}$$

6.3 EFFECTIVE STRESS CONCENTRATION FACTORS

In theory, the peak stress near a stress raiser would be K_t times larger than the nominal stress at the notched cross section. However, K_t is an ideal value based on linear elastic behavior and depends only on the proportions of the dimensions of the stress raiser and the notched part. For example, in case 2a, part I, Table 6-1, if h, D, and r were all multiplied by a common factor $n > 0$, the value of K_t would remain the same. In practice, a number of phenomena may act to mitigate the effects of stress concentration. Local plastic deformation, residual stress, notch radius, part size, temperature, material characteristics (e.g., grain size, work-hardening behavior), and load type (static, cyclic, or impact) may influence the extent to which the peak notch stress approaches the theoretical value of $K_t\,\sigma_{\text{nom}}$.

To deal with the various phenomena that influence stress concentration, the concepts of *effective stress concentration factor* and *notch sensitivity* have been introduced. The effective stress concentration factor is obtained experimentally.

The effective stress concentration factor of a specimen is defined to be the ratio of the stress calculated for the load at which structural damage is initiated in the specimen free of the stress raiser to the nominal stress corresponding to the load at which damage starts in the sample with the stress raiser. It is assumed that damage in the actual structure occurs when the maximum stress attains the same value in both cases. Similar to Eq. (6.2):

$$K_\sigma = \sigma_{\max}/\sigma_{\text{nom}} \tag{6.3}$$

The factor K_σ is now the effective stress concentration factor as determined by the experimental study of the specimen. See Ref. [6.1] for a more detailed discussion of K_σ.

For fatigue loading, the definition of experimentally determined effective stress concentration is

$$K_f = \frac{\text{fatigue strength without notch}}{\text{fatigue strength with notch}} \tag{6.4}$$

Factors determined by Eq. (6.4) should be regarded more as strength reduction factors than as quantities that correspond to an actual stress in the body. The fatigue strength (limit) is the maximum amplitude of fully reversed cyclic stress that a specimen can withstand for a given number of load cycles. For static conditions the stress at rupture is computed using strength-of-materials elastic formulas even though yielding may occur before rupture. If the tests are under bending or torsion loads, extreme fiber stress is used in the definition of K_σ and the stresses are computed using the formulas $\sigma = Mc/I$ and $\tau = Tr/J$ (Chapter 3).

No suitable experimental definition of the effective stress concentration factor in impact exists. Impact tests such as the *Charpy* or *Izod tests* (Chapter 4) measure the energy absorbed during the rupture of a notched specimen and do not yield information on stress levels.

When experimental information for a given member or load condition does not exist, the *notch sensitivity index q* provides a means of estimating the effects of stress concentration on strength. Effective stress concentration factors, which are less than the theoretical factor, are related to K_t by the equations

$$K_\sigma = 1 + q(K_t - 1) \tag{6.5}$$

$$K_f = 1 + q_f(K_t - 1) \tag{6.6}$$

A similar equation could be shown for impact loads using q_i as the notch sensitivity index. Often an explicit expression for the notch sensitivity index is given [e.g., $q_f = (K_f - 1)/(K_t - 1)$]. The notch sensitivity index can vary from 0 for complete insensitivity to notches to 1 for the full theoretical effect. Typical values of q are shown in Fig. 6-3.

Notch sensitivity in fatigue decreases as the notch radius decreases and as the grain size increases. A larger part will generally have greater notch sensitivity than a smaller part with proportionally similar dimensions. This variation is known as the *scale effect*. Larger notch radii result in lower stress gradients near the notch, and more material is subjected to higher stresses. Notch sensitivity in fatigue is therefore

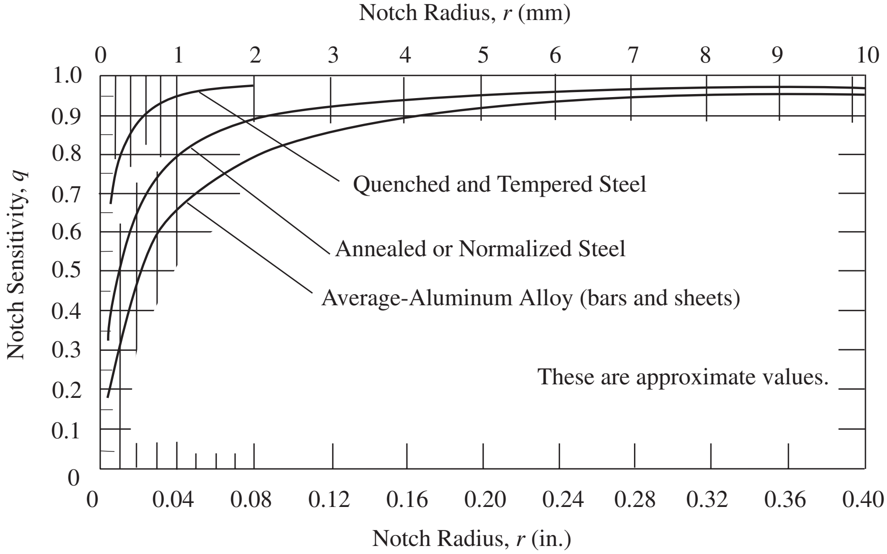


Figure 6-3: Fatigue notch sensitivity index.

increased. Because of the low sensitivity of small notch radii, the extremely high theoretical stress concentration factors predicted for very sharp notches and scratches are not actually realized. The notch sensitivity of quenched and tempered steels is higher than that of lower-strength, coarser-grained alloys. As a consequence, for notched members the strength advantage of high-grade steels over other materials may be lost.

Under static loading, notch sensitivity values are recommended [6.3] as $q = 0$ for ductile materials and q between 0.15 and 0.25 for hard, brittle metals. The notch insensitivity of ductile materials is caused by local plastic deformation at the notch tip. Under conditions that inhibit plastic slip, the notch sensitivity of a ductile metal may increase. Very low temperatures and high temperatures that cause viscous creep are two service conditions that may increase the notch sensitivity of some ductile metals. The notch sensitivity of cast iron is low for static loads ($q \approx 0$) because of the presence of internal stress raisers in the form of material inhomogeneities. These internal stress raisers weaken the material to such an extent that external notches have limited additional effect.

When a notched structural member is subjected to impact loads, the notch sensitivity may increase because the short duration of the load application does not permit the mitigating process of local slip to occur. Also, the small sections at stress raisers decrease the capacity of a member to absorb impact energy. For impact loads, values of notch sensitivity are recommended such as [6.3] q_i between 0.4 and 0.6 for ductile metals, $q_i = 1$ for hard, brittle materials, and $q_i = 0.5$ for cast irons. Reference [6.1] recommends using the full theoretical factor for brittle metals (including cast irons) for both static and impact loads because of the possibility of accidental shock loads being applied to a member during handling. The utilization of fracture mechanics to predict the brittle fracture of a flawed member under static, impact, and cyclic loads is treated in Chapter 7.

Neuber's Rule

Consider the stretched plate of Fig. 6-4. For nonlinear material behavior (Fig. 6-5), where local plastic deformation can occur near the hole, the previous stress concentration formulas may not apply. Neuber [6.4] established a rule that is useful beyond the elastic limit relating the effective stress and strain concentration factors to the theoretical stress concentration factor. Neuber's rule contends that the formula

$$K_\sigma \, K_\varepsilon = K_t^2 \tag{6.7}$$

applies to the three factors. This relation states that K_t is the geometric mean of K_σ and K_ε [i.e., $K_t = (K_\sigma \, K_\varepsilon)^{1/2}$]. Often, for fatigue, K_f replaces K_t. From the definition of effective stress concentration, $K_\sigma = \sigma_{\text{max}}/\sigma_{\text{nom}}$. Also, $K_\varepsilon = \varepsilon_{\text{max}}/\varepsilon_{\text{nom}}$ defines the effective strain concentration factor, where ε_{max} is the strain obtained from the material law (perhaps nonlinear) for the stress level σ_{max}. Using these relations in Eq. (6.7) yields

$$\sigma_{\text{max}} \, \varepsilon_{\text{max}} = K_t^2 \sigma_{\text{nom}} \, \varepsilon_{\text{nom}} \tag{6.8}$$

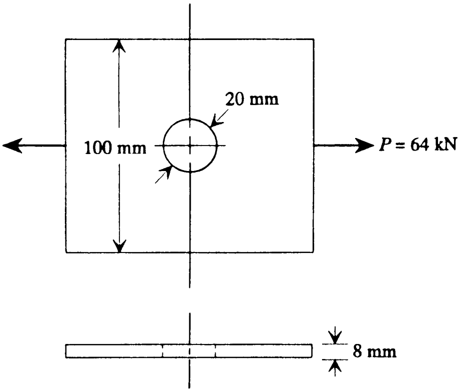


Figure 6-4: Tensile member with a hole.

Usually, K_t and σ_{nom} are known, and ε_{nom} can be found from the stress–strain curve for the material. Equation (6.8) therefore becomes

$$\sigma_{\max}\,\varepsilon_{\max} = C \tag{6.9}$$

where C is a known constant. Solving Eq. (6.9) simultaneously with the stress–strain relation, the values of maximum stress and strain are found, and the true (effective) stress concentration factor K_σ can then be determined. In this procedure the appropriate stress–strain curve must be known.

Neuber's rule was derived specifically for sharp notches in prismatic bars subjected to two-dimensional shear, but the rule has been applied as a useful approxima-

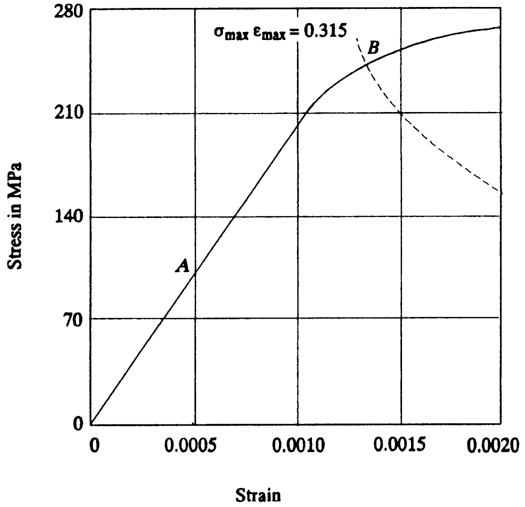


Figure 6-5: Stress–strain diagram for material of the tensile member of Fig. 6-4.

tion in other cases, especially those in which plane stress conditions exist. The rule has been shown to give poor results for circumferential grooves in shafts under axial tension [6.5].

Example 6.2 Tensile Member with a Circular Hole The member shown in Fig. 6-4 is subjected to an axial tensile load of 64 kN. The material from which the member is constructed has the stress–strain diagram of Fig. 6-5 for static tensile loading.

From Table 6-1, part II, case 2a, the theoretical stress concentration factor is computed using $d/D = \frac{20}{100}$, as

$$K_t = 3.0 - 3.140\left(\frac{20}{100}\right) + 3.667\left(\frac{20}{100}\right)^2 - 1.527\left(\frac{20}{100}\right)^3 = 2.51 \qquad (1)$$

The nominal stress is found using the net cross-sectional area:

$$\sigma_{\text{nom}} = \frac{P}{(D-d)t} = \frac{64}{(100-20)8}\left(\frac{10^3}{10^{-6}}\right) = 100 \text{ MPa} \qquad (2)$$

Based on elastic behavior, the peak stress $\sigma_{\max}$ at the edge of the hole would be

$$\sigma_{\max} = K_t\,\sigma_{\text{nom}} = (2.51)(100) = 251 \text{ MPa} \qquad (3)$$

This stress value, however, exceeds the yield point of the material. The actual peak stress and strain at the hole edge are found by using Neuber's rule. The nominal strain is read from the stress–strain curve; at $\sigma_{\text{nom}} = 100$ MPa, the strain is $\varepsilon_{\text{nom}} = 5 \times 10^{-4}$. The point $(\sigma_{\text{nom}}, \varepsilon_{\text{nom}})$ is point A in Fig. 6-5. Neuber's rule gives

$$\sigma_{\max}\,\varepsilon_{\max} = K_t^2\sigma_{\text{nom}}\varepsilon_{\text{nom}} = (2.51)^2(100)(5 \times 10^{-4}) = 0.315 \text{ MPa} \qquad (4)$$

The intersection of the curve $\sigma_{\max}\,\varepsilon_{\max} = 0.315$ with the stress–strain curve (point B in Fig. 6-5) yields a peak stress of $\sigma_{\max} = 243$ MPa and a peak strain of 13×10^{-4}. The effective stress concentration factor is

$$K_\sigma = \sigma_{\max}/\sigma_{\text{nom}} = 243/100 = 2.43 \qquad (5)$$

The effective strain concentration factor is

$$K_\varepsilon = \frac{13 \times 10^{-4}}{5 \times 10^{-4}} = 2.6 \qquad (6)$$

In the local strain approach to fatigue analysis, fatigue life is correlated with the strain history of a point, and knowledge of the true level of strain at the point is necessary. Neuber's rule enables the estimation of local strain levels without using complicated elastic–plastic finite-element analyses.

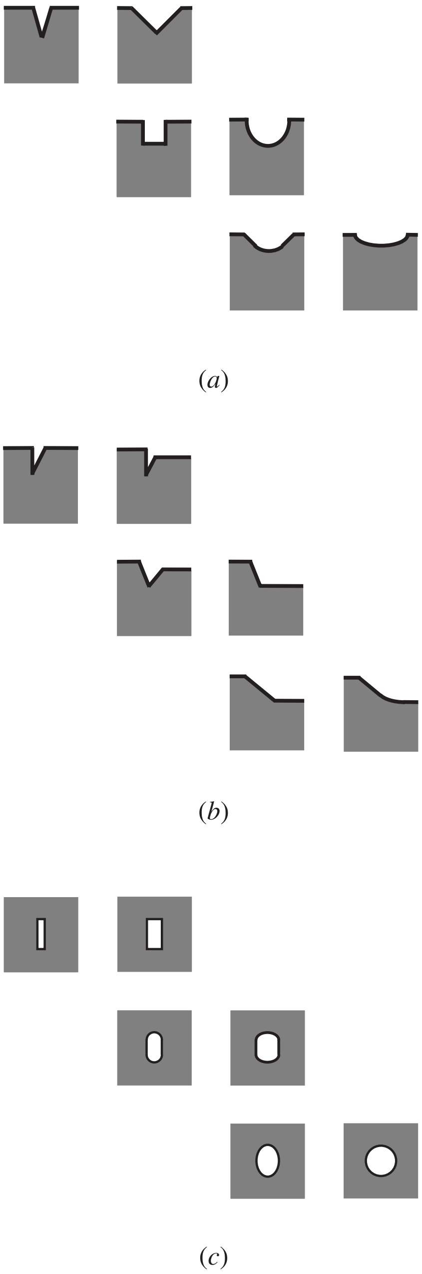


Figure 6-6: Reducing the effect of the stress concentration of notches and holes: (*a*) Notch shapes arranged in order of their effect on the stress concentration decreasing as you move from left to right and top to bottom; (*b*) asymmetric notch shapes, arranged in the same way as in (*a*); (*c*) holes, arranged in the same way as in (*a*).

6.4 DESIGNING TO MINIMIZE STRESS CONCENTRATION

A qualitative discussion of techniques for avoiding the detrimental effects of stress concentration is given by Leyer [6.6]. As a general rule, force should be transmitted from point to point as smoothly as possible. The lines connecting the force transmission path are sometimes called the *force* (or *stress*) *flow*, although it is arguable if *force flow* has a scientifically based definition. Sharp transitions in the direction of the force flow should be removed by smoothing contours and rounding notch roots. When stress raisers are necessitated by functional requirements, the raisers should be placed in regions of low nominal stress if possible. Figure 6-6 depicts forms of notches and holes in the order in which they cause stress concentration. Figure 6-7 shows how direction of stress flow affects the extent to which a notch causes stress concentration. The configuration in Fig. 6-7*b* has higher stress levels because of the sharp change in the direction of force flow.

When notches are necessary, removal of material near the notch can alleviate stress concentration effects. Figures 6-8 to 6-13 demonstrate instances where removal of material improves the strength of the member.

A type of stress concentration called an *interface notch* is commonly produced when parts are joined by welding. Figure 6-14 shows examples of interface notches and one way of mitigating the effect. The surfaces where the mating plates touch without weld metal filling, form what is, in effect, a sharp crack that causes stress concentration. Stress concentration also results from poor welding techniques that create small cracks in the weld material or burn pits in the base material.

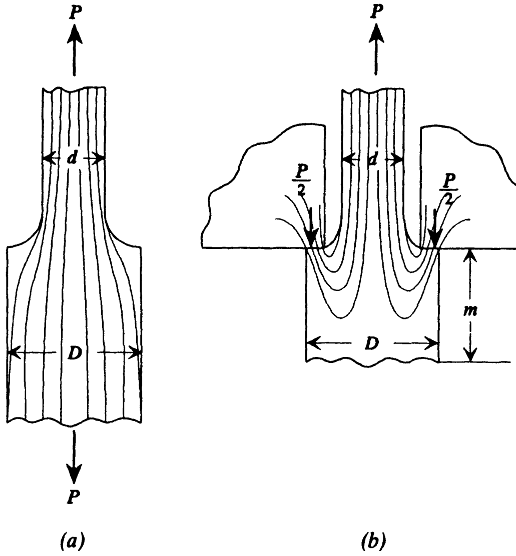


Figure 6-7: Two parts with the same shape (step in cross section) but differing stress flow patterns can give totally different notch effects and widely differing stress levels at the corner step: (*a*) stress flow is smooth; (*b*) sharp change in the stress flow direction causes high stress.

Figure 6-8: Guiding the lines of stress by means of notches that are not functionally essential is a useful method of reducing the detrimental effects of notches that cannot be avoided. These are termed relief notches. It is assumed here that the bearing surface of the step of (*a*) is needed functionally. Adding a notch as in (*b*) can reduce the hazardous effects of the corner of (*a*).

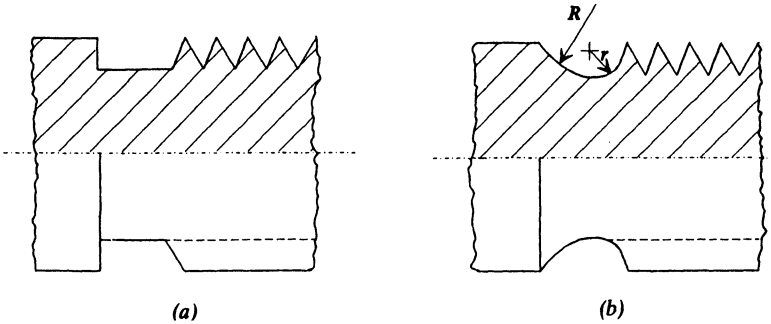


Figure 6-9: Relief notch where screw thread meets cylindrical body of bolt; (*a*) considerable stress concentration can occur at the step interface; (*b*) use of a smoother interface leads to relief of stress concentration.

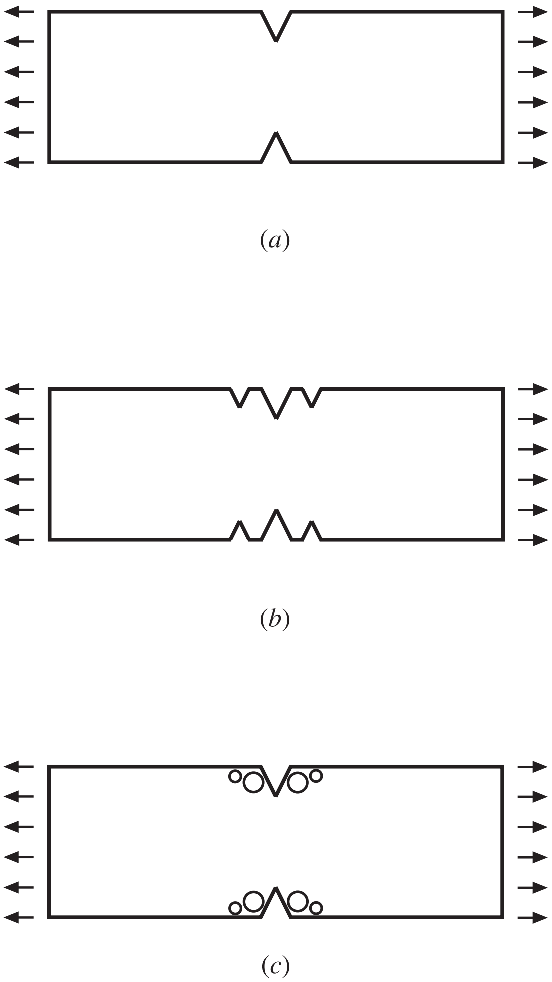
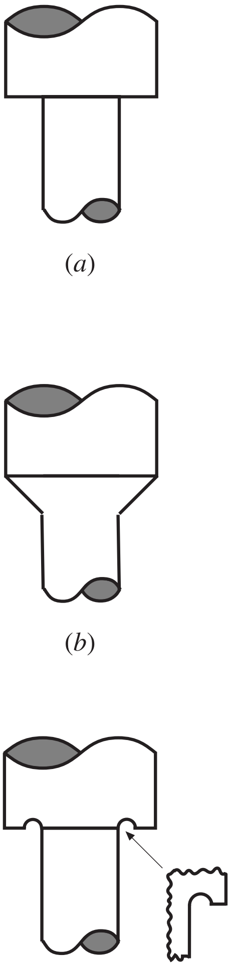


Figure 6-10: Alleviation of stress concentration by removal of material, a process that sometimes is relatively easy to machine. (*a*) It is assumed that a notch of the sort shown occurs. In both cases (*b*) and (*c*), the notch is retained and the stress concentration reduced.



(*a*)

(*b*)

(*c*)

Figure 6-11: Reduce the stress concentration in the stepped shaft of (*a*) by including material such as shown in (*b*). If this sort of modification is not possible, the undercut shoulder of (*c*) can help.

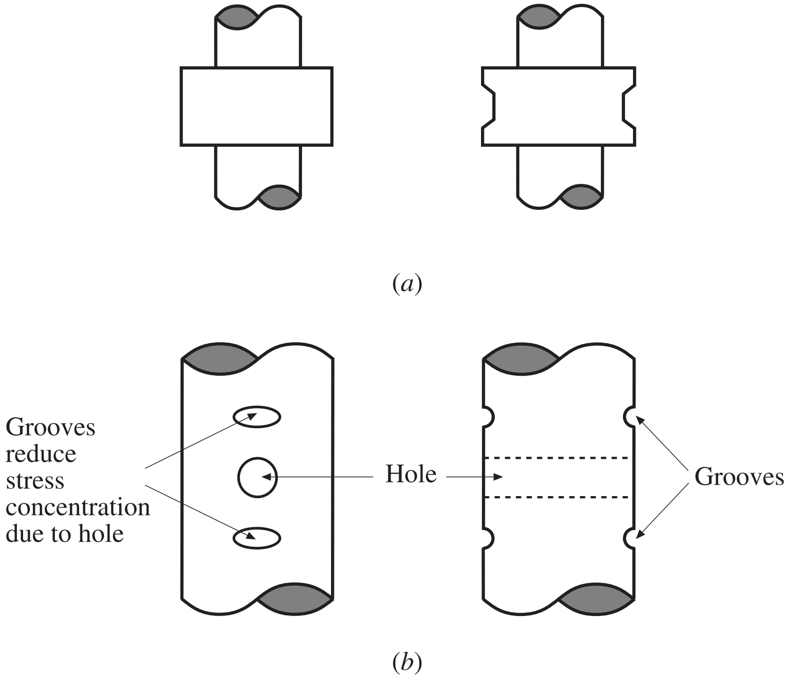


Figure 6-12: Removal of material can reduce stress concentration, for example, in bars with collars and holes. (*a*) The bar on the right with the narrowed collar will lead to reduced stress concentration relative to the bar on the left. (*b*) Grooves near a hole can reduce the stress concentration around the hole.

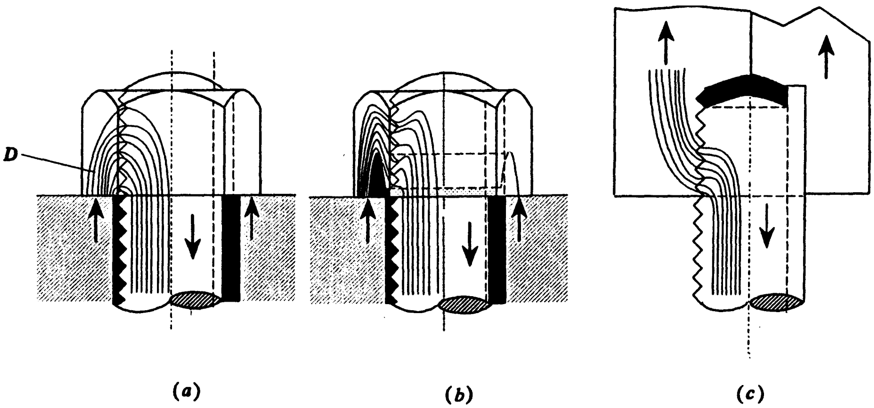


Figure 6-13: Nut designs. These are most important under fatigue loading. From Ref. [6.1], with permission. (*a*) Standard bolt and nut combination. The force flow near the top of the nut is sparse, but in area *D* the stress flow density is very high. (*b*) Nut with a lip. The force flow on the inner side of the lip is in the same direction as in the bolt and the force flow is more evenly distributed for the whole nut than for case (*a*). The peak stress is relieved. (*c*) "Force flow" is not reversed at all. Thus fatigue strength here is significantly higher than for the other cases.

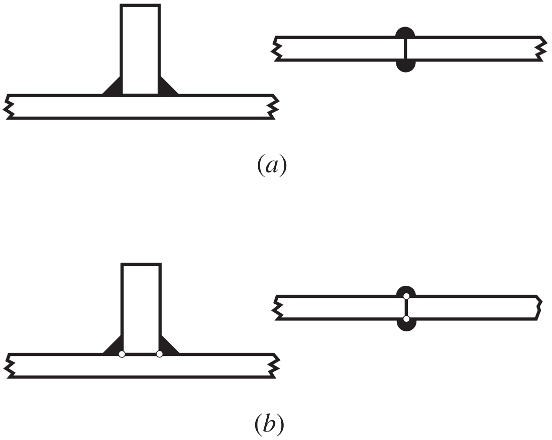


Figure 6-14: The typical welding joints of (*a*) can be improved by boring out corners as shown in (*b*).

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Tables

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- III. Fillets 298
- IV. Miscellaneous Elements 301

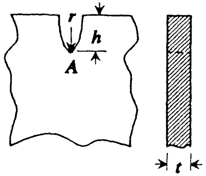
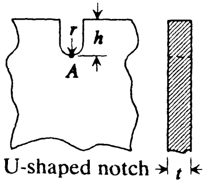
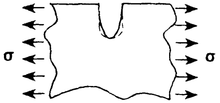
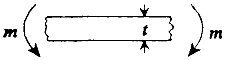
TABLE 6-1 STRESS CONCENTRATION FACTORS[a]

Notation

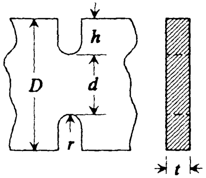
K_t	Theoretical stress concentration factor in elastic range	σ_{nom}	Nominal normal stress defined for each case (F/L^2)
σ	Applied stress (F/L^2)	$\sigma_{\max}$	Maximum normal stress at stress raiser (F/L^2)
P	Applied axial force (F)	τ_{nom}	Nominal shear stress defined for each case (F/L^2)
M	Applied moment (FL)		
m_1, m_2, m	Applied moment per unit length (FL/L)	$\tau_{\max}$	Maximum shear stress at stress raiser (F/L^2)
T	Applied torque (FL)		

Refer to figures for the geometries of the specimens.

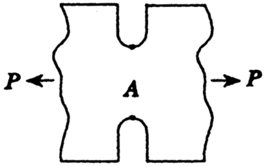
I. Notches and Grooves

Type of Stress Raiser	Loading Condition	Stress Concentration Factor
1. Elliptical or U-shaped notch in semi-infinite plate  **Elliptical notch**  **U-shaped notch**	a. Uniaxial tension 	$\sigma_{\max} = \sigma_A = K_t\sigma$ $K_t = 0.855 + 2.21\sqrt{h/r}$ for $1 \le h/r \le 361$
	b. Transverse bending 	Elliptical notch only, $\nu = 0.3$ and when $h/t \to \infty$, $\sigma_{\max} = \sigma_A = K_t\sigma, \sigma = 6m/t^2$ $K_t = 0.998 + 0.790\sqrt{h/r}$ for $0 \le h/r \le 7$

2.
Opposite single U-shaped notches in finite-width plate



a. Axial tension



$$\sigma_{\max} = \sigma_A = K_t \sigma_{\text{nom}}, \qquad \sigma_{\text{nom}} = P/td$$

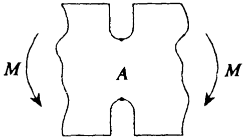
$$K_t = C_1 + C_2 \frac{2h}{D} + C_3 \left(\frac{2h}{D}\right)^2 + C_4 \left(\frac{2h}{D}\right)^3$$

	$0.1 \le h/r < 2.0$	$2.0 \le h/r \le 50.0$
C_1	$0.955 + 2.169\sqrt{h/r} - 0.081h/r$	$1.037 + 1.991\sqrt{h/r} + 0.002h/r$
C_2	$-1.557 - 4.046\sqrt{h/r} + 1.032h/r$	$-1.886 - 2.181\sqrt{h/r} - 0.048h/r$
C_3	$4.013 + 0.424\sqrt{h/r} - 0.748h/r$	$0.649 + 1.086\sqrt{h/r} + 0.142h/r$
C_4	$-2.461 + 1.538\sqrt{h/r} - 0.236h/r$	$1.218 - 0.922\sqrt{h/r} - 0.086h/r$

for semicircular notch ($h/r = 1.0$)

$$K_t = 3.065 - 3.472 \left(\frac{2h}{D}\right) + 1.009 \left(\frac{2h}{D}\right)^2 + 0.405 \left(\frac{2h}{D}\right)^3$$

b. In-plane bending



$$\sigma_{\max} = \sigma_A = K_t \sigma_{\text{nom}}, \qquad \sigma_{\text{nom}} = 6M/d^2t$$

$$K_t = C_1 + C_2 \frac{2h}{D} + C_3 \left(\frac{2h}{D}\right)^2 + C_4 \left(\frac{2h}{D}\right)^3$$

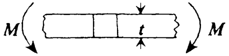
	$0.1 \le h/r < 2.0$	$2.0 \le h/r \le 50.0$
C_1	$1.024 + 2.092\sqrt{h/r} - 0.051h/r$	$1.113 + 1.957\sqrt{h/r}$
C_2	$-0.630 - 7.194\sqrt{h/r} + 1.288h/r$	$-2.579 - 4.017\sqrt{h/r} - 0.013h/r$
C_3	$2.117 + 8.574\sqrt{h/r} - 2.160h/r$	$4.100 + 3.922\sqrt{h/r} + 0.083h/r$
C_4	$-1.420 - 3.494\sqrt{h/r} + 0.932h/r$	$-1.528 - 1.893\sqrt{h/r} - 0.066h/r$

for semicircular notch ($h/r = 1.0$)

$$K_t = 3.065 - 6.637 \left(\frac{2h}{D}\right) + 8.229 \left(\frac{2h}{D}\right)^2 - 3.636 \left(\frac{2h}{D}\right)^3$$

TABLE 6-1 (continued) STRESS CONCENTRATION FACTORS: Notches and Grooves

c. Transverse bending



$\sigma_{\max} = \sigma_A = K_t \sigma_{\text{nom}}, \qquad \sigma_{\text{nom}} = 6M/t^2 d$

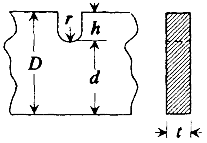
$$K_t = C_1 + C_2 \frac{2h}{D} + C_3 \left(\frac{2h}{D}\right)^2 + C_4 \left(\frac{2h}{D}\right)^3$$

	$0.1 \le h/r \le 5.0$ and h/t is large
C_1	$1.041 + 0.839\sqrt{h/r} + 0.014\, h/r$
C_2	$-1.239 - 1.663\sqrt{h/r} + 0.118\, h/r$
C_3	$3.370 - 0.758\sqrt{h/r} + 0.434\, h/r$
C_4	$-2.162 + 1.582\sqrt{h/r} - 0.606\, h/r$

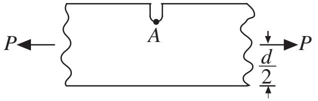
for semicircular notch $(h/r = 1.0)$

$$K_t = 1.894 - 2.784\left(\frac{2h}{D}\right) + 3.046\left(\frac{2h}{D}\right)^2 - 1.186\left(\frac{2h}{D}\right)^3$$

3.
Single U-shaped notch on one side in finite-width plate



a. Axial tension



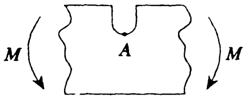
$\sigma_{\max} = \sigma_A = K_t \sigma_{\text{nom}}, \qquad \sigma_{\text{nom}} = P/td$

$$K_t = C_1 + C_2\left(\frac{h}{D}\right) + C_3\left(\frac{h}{D}\right)^2 + C_4\left(\frac{h}{D}\right)^3$$

	$0.5 \le h/r < 2.0$	$2.0 \le h/r \le 20.0$
C_1	$0.907 + 2.125\sqrt{h/r} + 0.023h/r$	$0.953 + 2.136\sqrt{h/r} - 0.005h/r$
C_2	$0.710 - 11.289\sqrt{h/r} + 1.708h/r$	$-3.255 - 6.281\sqrt{h/r} + 0.068h/r$
C_3	$-0.672 + 18.754\sqrt{h/r} - 4.046h/r$	$8.203 + 6.893\sqrt{h/r} + 0.064h/r$
C_4	$0.175 - 9.759\sqrt{h/r} + 2.365h/r$	$-4.851 - 2.793\sqrt{h/r} - 0.128h/r$

for semicircular notch $(h/r = 1.0)$

$$K_t = 3.065 - 8.871\left(\frac{h}{D}\right) + 14.036\left(\frac{h}{D}\right)^2 - 7.219\left(\frac{h}{D}\right)^3$$

<table>
<tr>
<td></td>
<td>b. In-plane bending
 </td>
<td>

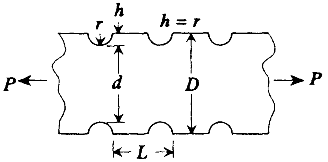
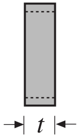
$\sigma_{\max} = \sigma_A = K_t \sigma_{\text{nom}}, \qquad \sigma_{\text{nom}} = 6M/td^2$

$K_t = C_1 + C_2\left(\frac{h}{D}\right) + C_3\left(\frac{h}{D}\right)^2 + C_4\left(\frac{h}{D}\right)^3$

<table>
<tr><th></th><th>$0.5 \le h/r \le 2.0$</th><th>$2.0 \le h/r \le 20.0$</th></tr>
<tr><td>C_1</td><td>$1.795 + 1.481h/r - 0.211(h/r)^2$</td><td>$2.966 + 0.502h/r - 0.009(h/r)^2$</td></tr>
<tr><td>C_2</td><td>$-3.544 - 3.677h/r + 0.578(h/r)^2$</td><td>$-6.475 - 1.126h/r + 0.019(h/r)^2$</td></tr>
<tr><td>C_3</td><td>$5.459 + 3.691h/r - 0.565(h/r)^2$</td><td>$8.023 + 1.253h/r - 0.020(h/r)^2$</td></tr>
<tr><td>C_4</td><td>$-2.678 - 1.531h/r + 0.205(h/r)^2$</td><td>$-3.572 - 0.634h/r + 0.010(h/r)^2$</td></tr>
</table>

for semicircular notch $(h/r = 1.0)$

$K_t = 3.065 - 6.643\left(\frac{h}{D}\right) + 0.205\left(\frac{h}{D}\right)^2 - 4.004\left(\frac{h}{D}\right)^3$

</td>
</tr>
<tr>
<td>4.
Multiple opposite semicircular notches in finite-width plate
 </td>
<td>Axial tension
 </td>
<td>

$\sigma_{\max} = K_t \sigma_{\text{nom}}, \qquad \sigma_{\text{nom}} = P/td$

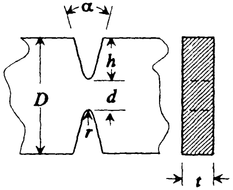
$K_t = C_1 + C_2\left(\frac{2r}{L}\right) + C_3\left(\frac{2r}{L}\right)^2 + C_4\left(\frac{2r}{L}\right)^3$

<table>
<tr><th></th><th>$2r/D \le 0.4, \qquad 0 \le 2r/L \le 1.0$</th></tr>
<tr><td>C_1</td><td>$3.1055 - 3.4287\left(\frac{2r}{D}\right) + 0.8522\left(\frac{2r}{D}\right)^2$</td></tr>
<tr><td>C_2</td><td>$-1.4370 + 10.5053\left(\frac{2r}{D}\right) - 8.7547\left(\frac{2r}{D}\right)^2 - 19.6273\left(\frac{2r}{D}\right)^3$</td></tr>
<tr><td>C_3</td><td>$-1.6753 - 14.0851\left(\frac{2r}{D}\right) + 43.6575\left(\frac{2r}{D}\right)^2$</td></tr>
<tr><td>C_4</td><td>$1.7207 + 5.7974\left(\frac{2r}{D}\right) - 27.7463\left(\frac{2r}{D}\right)^2 + 6.0444\left(\frac{2r}{D}\right)^3$</td></tr>
</table>

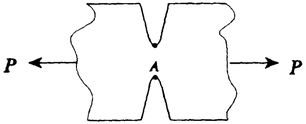
</td>
</tr>
</table>

TABLE 6-1 (continued) STRESS CONCENTRATION FACTORS: Notches and Grooves

5.
Opposite single V-shaped notches in finite-width plate



Axial tension



$\sigma_{\max} = \sigma_A = K_t \sigma_{\text{nom}}, \qquad \sigma_{\text{nom}} = P/td$

For $2h/D = 0.398$ and $\alpha < 90°$,

$2h/D = 0.667$ and $\alpha < 60°$:

$$K_t = K_{tu}$$

K_{tu} is the stress concentration factor for U-shaped notch and α is notch angle in degrees. Otherwise,

$K_t = C_1 + C_2\sqrt{K_{tu}} + C_3 K_{tu}$.

	$2h/D = 0.398,\ 90° \leq \alpha \leq 150°,\ 1.6 \leq K_{tu} \leq 3.5$
C_1	$5.294 - 0.1225\alpha + 0.000523\alpha^2$
C_2	$-5.0002 + 0.1171\alpha - 0.000434\alpha^2$
C_3	$1.423 - 0.01197\alpha - 0.000004\alpha^2$

	$2h/D = 0.667,\ 60° \leq \alpha \leq 150°,\ 1.6 \leq K_{tu} \leq 2.8$
C_1	$-10.01 + 0.1534\alpha - 0.000647\alpha^2$
C_2	$13.60 - 0.2140\alpha + 0.000973\alpha^2$
C_3	$-3.781 + 0.07873\alpha - 0.000392\alpha^2$

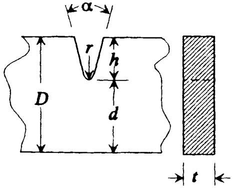
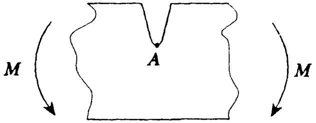
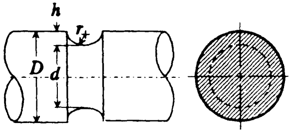
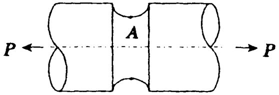
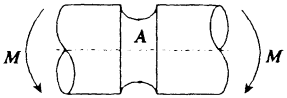
<table>
<tr>
<td>6.
Single V-shaped notch
on one side
 </td>
<td>In-plane bending
 </td>
<td>$\sigma_{\max} = \sigma_A = K_t\sigma_{\text{nom}}, \qquad \sigma_{\text{nom}} = 6M/td^2$
For $\alpha \le 90°$,
$K_t = K_{tu}$
For $90° < \alpha \le 150°$ and $0.5 \le h/r \le 4.0$,
$K_t = 1.11K_{tu} - \left[-0.0159 + 0.2243\left(\frac{\alpha}{150}\right) - 0.4293\left(\frac{\alpha}{150}\right)^2 + 0.3609\left(\frac{\alpha}{150}\right)^3\right]K_{tu}^2$
K_{tu} is the stress concentration factor for U notch, case 3b, and α is notch angle in degrees.</td>
</tr>
<tr>
<td>7.
U-shaped circumferential
groove in circular shaft
 </td>
<td>a. Axial tension
 </td>
<td>$\sigma_{\max} = \sigma_A = K_t\sigma_{\text{nom}}, \qquad \sigma_{\text{nom}} = 4P/\pi d^2$
$K_t = C_1 + C_2\frac{2h}{D} + C_3\left(\frac{2h}{D}\right)^2 + C_4\left(\frac{2h}{D}\right)^3$
<table>
<tr><th></th><th>$0.1 \le h/r < 2.0$</th><th>$2.0 \le h/r \le 50.0$</th></tr>
<tr><td>C_1</td><td>$0.89 + 2.208\sqrt{h/r} - 0.094h/r$</td><td>$1.037 + 1.967\sqrt{h/r} + 0.002h/r$</td></tr>
<tr><td>C_2</td><td>$-0.923 - 6.678\sqrt{h/r} + 1.638h/r$</td><td>$-2.679 - 2.980\sqrt{h/r} - 0.053h/r$</td></tr>
<tr><td>C_3</td><td>$2.893 + 6.448\sqrt{h/r} - 2.516h/r$</td><td>$3.090 + 2.124\sqrt{h/r} + 0.165h/r$</td></tr>
<tr><td>C_4</td><td>$-1.912 - 1.944\sqrt{h/r} + 0.963h/r$</td><td>$-0.424 - 1.153\sqrt{h/r} - 0.106h/r$</td></tr>
</table>
for semicircular groove ($h/r = 1.0$)
$K_t = 3.004 - 5.963\left(\frac{2h}{D}\right) + 6.836\left(\frac{2h}{D}\right)^2 - 2.893\left(\frac{2h}{D}\right)^3$</td>
</tr>
</table>

TABLE 6-1 (continued) STRESS CONCENTRATION FACTORS: Notches and Grooves

b. Bending



$$\sigma_{\max} = \sigma_A = K_t \sigma_{\text{nom}}, \qquad \sigma_{\text{nom}} = 32\ M/\pi d^3$$

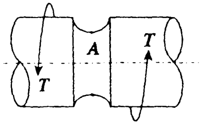
$$K_t = C_1 + C_2 \frac{2h}{D} + C_3 \left(\frac{2h}{D}\right)^2 + C_4 \left(\frac{2h}{D}\right)^3$$

	$0.25 \le h/r < 2.0$	$2.0 \le h/r \le 50.0$
C_1	$0.594 + 2.958\sqrt{h/r} - 0.520h/r$	$0.965 + 1.926\sqrt{h/r}$
C_2	$0.422 - 10.545\sqrt{h/r} + 2.692h/r$	$-2.773 - 4.414\sqrt{h/r} - 0.017h/r$
C_3	$0.501 + 14.375\sqrt{h/r} - 4.486h/r$	$4.785 + 4.681\sqrt{h/r} + 0.096h/r$
C_4	$-0.613 - 6.573\sqrt{h/r} + 2.177h/r$	$-1.995 - 2.241\sqrt{h/r} - 0.074h/r$

for semicircular groove ($h/r = 1.0$)

$$K_t = 3.032 - 7.431 \left(\frac{2h}{D}\right) + 10.390 \left(\frac{2h}{D}\right)^2 - 5.009 \left(\frac{2h}{D}\right)^3$$

c. Torsion



$$\tau_{\max} = \tau_A = K_t \tau_{\text{nom}}, \qquad \tau_{\text{nom}} = 16T/\pi d^3$$

$$K_t = C_1 + C_2 \frac{2h}{D} + C_3 \left(\frac{2h}{D}\right)^2 + C_4 \left(\frac{2h}{D}\right)^3$$

	$0.25 \le h/r < 2.0$	$2.0 \le h/r \le 50.0$
C_1	$0.966 + 1.056\sqrt{h/r} - 0.022h/r$	$1.089 + 0.924\sqrt{h/r} + 0.018h/r$
C_2	$-0.192 - 4.037\sqrt{h/r} + 0.674h/r$	$-1.504 - 2.141\sqrt{h/r} - 0.047h/r$
C_3	$0.808 + 5.321\sqrt{h/r} - 1.231h/r$	$2.486 + 2.289\sqrt{h/r} + 0.091h/r$
C_4	$-0.567 - 2.364\sqrt{h/r} + 0.566h/r$	$-1.056 - 1.104\sqrt{h/r} - 0.059h/r$

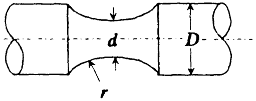
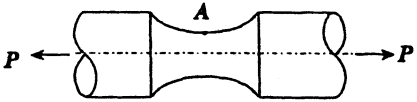
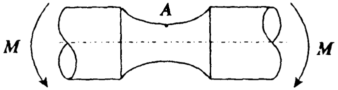
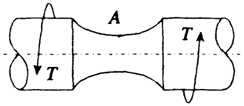
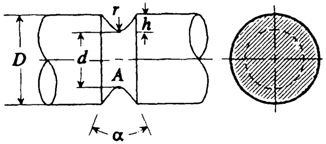
<table>
<tr>
<td rowspan="3">8.
Large circumferential groove in circular shaft
 </td>
<td>a. Axial tension
 </td>
<td>$\sigma_{\max} = \sigma_A = K_t\sigma_{\text{nom}}, \qquad \sigma_{\text{nom}} = 4P/\pi d^2$
$K_t = C_1 + C_2(r/d) + C_3(r/d)^2$
<table>
<tr><th></th><th>$0.3 \le r/d \le 1.0, \qquad 1.005 \le D/d \le 1.10$</th></tr>
<tr><td>C_1</td><td>$-81.39 + 153.10(D/d) - 70.49(D/d)^2$</td></tr>
<tr><td>C_2</td><td>$119.64 - 221.81(D/d) + 101.93(D/d)^2$</td></tr>
<tr><td>C_3</td><td>$-57.88 + 107.33(D/d) - 49.34(D/d)^2$</td></tr>
</table></td>
</tr>
<tr>
<td>b. Bending
 </td>
<td>$\sigma_{\max} = \sigma_A = K_t\sigma_{\text{nom}}, \qquad \sigma_{\text{nom}} = 32M/\pi d^3$
$K_t = C_1 + C_2(r/d) + C_3(r/d)^2$
<table>
<tr><th></th><th>$0.3 \le r/d \le 1.0, \qquad 1.005 \le D/d < 1.10$</th></tr>
<tr><td>C_1</td><td>$-39.58 + 73.22(D/d) - 32.46(D/d)^2$</td></tr>
<tr><td>C_2</td><td>$-9.477 + 29.41(D/d) - 20.13(D/d)^2$</td></tr>
<tr><td>C_3</td><td>$82.46 - 166.96(D/d) + 84.58(D/d)^2$</td></tr>
</table></td>
</tr>
<tr>
<td>c. Torsion
 </td>
<td>$\tau_{\max} = \tau_A = K_t\tau_{\text{nom}}, \qquad \tau_{\text{nom}} = 16T/\pi d^3$
$K_t = C_1 + C_2(r/d) + C_3(r/d)^2$
<table>
<tr><th></th><th>$0.3 \le r/d \le 1, \qquad 1.005 \le D/d < 1.10$</th></tr>
<tr><td>C_1</td><td>$-35.16 + 67.57(D/d) - 31.28(D/d)^2$</td></tr>
<tr><td>C_2</td><td>$79.13 - 148.37(D/d) + 69.09(D/d)^2$</td></tr>
<tr><td>C_3</td><td>$-50.34 + 94.67(D/d) - 44.26(D/d)^2$</td></tr>
</table></td>
</tr>
</table>

TABLE 6-1 (continued) STRESS CONCENTRATION FACTORS: Notches and Grooves

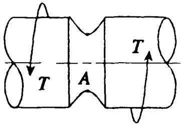
<table>
<tr>
<td>

9.
V-shaped groove in circular shaft



</td>
<td>

Torsion



</td>
<td>

$\tau_{\max} = \tau_A = K_t \tau_{\text{nom}}, \qquad \tau_{\text{nom}} = 16T/\pi d^3$

K_{tu} = stress concentration factor for U-shaped groove ($\alpha = 0$), case 7c

<table>
<tr><th></th><th>$K_t = C_1 + C_2\sqrt{K_{tu}} + C_3 K_{tu}$</th></tr>
<tr><td>C_1</td><td>$0.2026\sqrt{\alpha} - 0.06620\alpha + 0.00281\alpha\sqrt{\alpha}$</td></tr>
<tr><td>C_2</td><td>$-0.2226\sqrt{\alpha} + 0.07814\alpha - 0.002477\alpha\sqrt{\alpha}$</td></tr>
<tr><td>C_3</td><td>$1 + 0.0298\sqrt{\alpha} - 0.01485\alpha - 0.000151\alpha\sqrt{\alpha}$</td></tr>
</table>

where α is in degrees.

For $0^\circ \leq \alpha \leq 90^\circ$, K_t is independent of r/d; for $90^\circ \leq \alpha \leq 125^\circ$, K_t is applicable only if $r/d \leq 0.01$.

</td>
</tr>
</table>

II. Holes

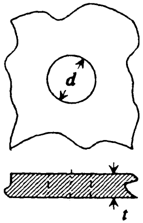
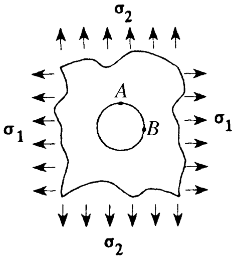
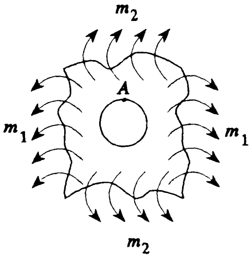
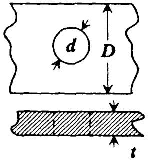
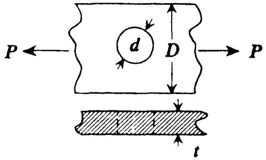
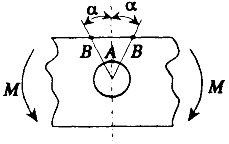
Type of Stress Raiser	Loading Condition	Stress Concentration Factor
1. Single circular hole in infinite plate 	a. In-plane normal stress 	(1) Uniaxial tension ($\sigma_2 = 0$) $\sigma_{\max} = K_t\sigma_1$ $\sigma_A = 3\sigma_1$ or $K_t = 3$ $\sigma_B = -\sigma_1$ or $K_t = -1$ (2) Biaxial tension $K_t = 3 - \sigma_2/\sigma_1$ for $-1 \le \sigma_2/\sigma_1 \le 1$ For $\sigma_2 = \sigma_1,\ \sigma_A = \sigma_B = 2\sigma_1$ or $K_t = 2$ For $\sigma_2 = -\sigma_1$ (pure shear stress), $\sigma_A = -\sigma_B = 4\sigma_1$ or $K_t = 4$
	b. Transverse bending 	$\sigma_{\max} = K_t\sigma, \qquad \sigma = 6m/t^2, \qquad \nu = 0.3$ (1) Simple bending ($m_1 = m,\ m_2 = 0$) For $0 \le d/t \le 7.0$, $\sigma_{\max} = \sigma_A$ $K_t = 3.000 - 0.947\sqrt{d/t} + 0.192d/t$ (2) Cylindrical bending ($m_1 = m,\ m_2 = \nu m$) For $0 \le d/t \le 7.0$, $\sigma_{\max} = \sigma_A$ $K_t = 2.700 - 0.647\sqrt{d/t} + 0.129d/t$ (3) Isotropic bending ($m_1 = m_2 = m$), $\sigma_{\max} = \sigma_A$ $K_t = 2$ (independent of d/t)
	c. Twisting moment (see preceding figure and definitions)	$\sigma_{\max} = K_t\sigma, \qquad \sigma = 6m/t^2$ $m_1 = m, \qquad m_2 = -m, \qquad \nu = 0.3$ For $0 \le d/t \le 7.0$, $K_t = 4.000 - 1.772\sqrt{d/t} + 0.341d/t$

TABLE 6-1 (continued) STRESS CONCENTRATION FACTORS: Holes

2. Central single circular hole in finite-width plate 	a. Axial tension 	$\sigma_{\max} = \sigma_A = K_t \sigma_{\text{nom}}, \qquad \sigma_{\text{nom}} = P/[t(D-d)]$ $K_t = 3.000 - 3.140(d/D) + 3.667(d/D)^2 - 1.527(d/D)^3$ for $0 \le d/D \le 1$
	b. In-plate bending 	(1) At edge of hole, $\sigma_{\max} = \sigma_A = K_t \sigma_{\text{nom}}, \qquad \sigma_{\text{nom}} = 6Md/(D^3 - d^3)t$ $K_t = 2$ (independent of d/D) (2) At edge of plate, $\sigma_{\max} = \sigma_B = K_t \sigma_{\text{nom}}, \qquad \sigma_{\text{nom}} = 6MD/(D^3 - d^3)t$ $K_t = 2d/D(\alpha = 30°)$

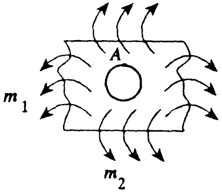
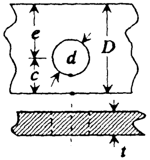
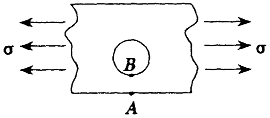
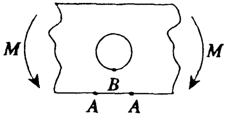
	c. Transverse bending  	$\sigma_{\max} = \sigma_A = K_t\sigma_{\text{nom}}, \qquad \sigma_{\text{nom}} = 6mD/(D-d)t^2$ For $0 \le d/D \le 0.3,\ \nu = 0.3$ and $1 \le d/t \le 7$ (1) Simple bending ($m_1 = m,\ m_2 = 0$) $K_t = \left[1.793 + \frac{0.131}{d/t} + \frac{2.052}{(d/t)^2} - \frac{1.019}{(d/t)^3}\right] \times \left[1 - 1.04\left(\frac{d}{D}\right) + 1.22\left(\frac{d}{D}\right)^2\right]$ (2) Cylindrical bending ($m_1 = m,\ m_2 = \nu m$) $K_t = \left[1.856 + \frac{0.317}{d/t} + \frac{0.942}{(d/t)^2} - \frac{0.415}{(d/t)^3}\right] \times \left[1 - 1.04\left(\frac{d}{D}\right) + 1.22\left(\frac{d}{D}\right)^2\right]$
3. Eccentric circular hole in finite-width plane  	a. Axial tension  	Stress on section AB is $\sigma_{\text{nom}} = \frac{\sigma\sqrt{1-(d/2c)^2}}{1-d/2c}\,\frac{1-c/D}{1-(c/d)\left[2-\sqrt{1-(d/2c)^2}\right]}$ $\sigma_{\max} = \sigma_B = K_t\sigma_{\text{nom}}$ $K_t = 3.000 - 3.140\left(\frac{d}{2c}\right) + 3.667\left(\frac{d}{2c}\right)^2 - 1.527\left(\frac{d}{2c}\right)^3$

TABLE 6-1 (continued) STRESS CONCENTRATION FACTORS: Holes

<table>
<tr>
<td></td>
<td>b. In-plane bending
 </td>
<td>$\sigma_{\max} = \max(\sigma_A, \sigma_B)$
$\sigma_B = K_{t_B}\sigma_{\text{nom}}, \qquad \sigma_{\text{nom}} = 6M/D^2t$
$K_{t_B} = C_1 + C_2\frac{c}{e} + C_3\left(\frac{c}{e}\right)^2$
<table>
<tr><td></td><td>$0 \le d/2c \le 0.5, \qquad 0 \le c/e \le 1.0$</td></tr>
<tr><td>C_1</td><td>$3.000 - 0.631(d/2c) + 4.007(d/2c)^2$</td></tr>
<tr><td>C_2</td><td>$-5.083 + 4.067(d/2c) - 2.795(d/2c)^2$</td></tr>
<tr><td>C_3</td><td>$2.114 - 1.682(d/2c) - 0.273(d/2c)^2$</td></tr>
</table>
$\sigma_A = K_{t_A}\sigma_{\text{nom}}, \qquad \sigma_{\text{nom}} = 6M/D^2t$
$K_{t_A} = C_1' + C_2'\frac{c}{e} + C_3'\left(\frac{c}{e}\right)^2$
<table>
<tr><td>C_1'</td><td>$1.0286 - 0.1638(d/2c) + 2.702(d/2c)^2$</td></tr>
<tr><td>C_2'</td><td>$-0.05863 - 0.1335(d/2c) - 1.8747(d/2c)^2$</td></tr>
<tr><td>C_3'</td><td>$0.18883 - 0.89219(d/2c) + 1.5189(d/2c)^2$</td></tr>
</table>
</td>
</tr>
<tr>
<td>4.
Two equal circular holes in infinite plate</td>
<td>a. Uniaxial tension parallel to row of holes ($\sigma_1 = \sigma,\ \sigma_2 = 0$)</td>
<td>$\sigma_{\max} = K_t\sigma$ for $0 \le d/L \le 1$
$K_t = 3.000 - 0.712\left(\frac{d}{L}\right) + 0.271\left(\frac{d}{L}\right)^2$</td>
</tr>
</table>

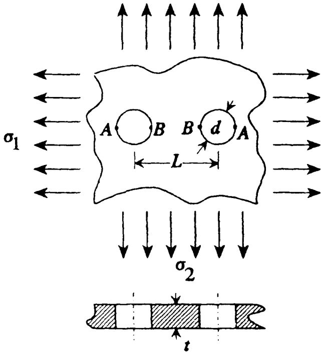
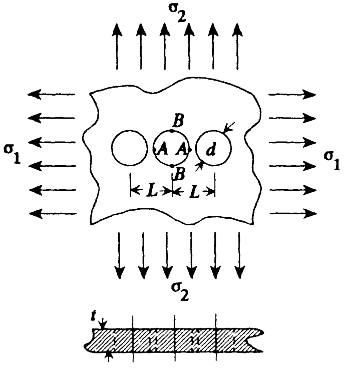
	b. Uniaxial tension normal to row of holes ($\sigma_2 = \sigma, \sigma_1 = 0$)	$\sigma_{\max} = \sigma_B = K_t \sigma_{\text{nom}}, \qquad \sigma_{\text{nom}} = \frac{\sigma\sqrt{1-(d/L)^2}}{1-d/L}$ $K_t = 3.0000 - 3.0018\left(\frac{d}{L}\right) + 1.0099\left(\frac{d}{L}\right)^2$ for $0 \le d/L \le 1$
	c. Biaxial tension ($\sigma_1 = \sigma_2 = \sigma$)	$\sigma_{\max} = \sigma_B = K_t \sigma_{\text{nom}}, \qquad \sigma_{\text{nom}} = \frac{\sigma\sqrt{1-(d/L)^2}}{1-d/L}$ $K_t = 2.000 - 2.119\left(\frac{d}{L}\right) + 2.493\left(\frac{d}{L}\right)^2 - 1.372\left(\frac{d}{L}\right)^3$ for $0 \le d/L \le 1$

TABLE 6-1 (continued) STRESS CONCENTRATION FACTORS: Holes

5. Single row of circular holes in infinite plate  	a. Uniaxial tension normal to row of holes $(\sigma_1 = 0,\ \sigma_2 = \sigma)$	$\sigma_{\max} = \sigma_B = K_t \sigma$ $K_t = 3.0000 - 0.9916\left(\frac{d}{L}\right) - 2.5899\left(\frac{d}{L}\right)^2 + 2.2613\left(\frac{d}{L}\right)^3$ for $0 \leq d/L \leq 1$
	b. Uniaxial tension parallel to row of holes $(\sigma_1 = \sigma,\ \sigma_2 = 0)$	$\sigma_{\max} = \sigma_A = K_t \sigma_{\text{nom}}, \qquad \sigma_{\text{nom}} = \sigma/(1 - d/L)$ $K_t = 3.000 - 3.095\left(\frac{d}{L}\right) + 0.309\left(\frac{d}{L}\right)^2 + 0.786\left(\frac{d}{L}\right)^3$ for $0 \leq d/L \leq 1$
	c. Biaxial tension $(\sigma_1 = \sigma_2 = \sigma)$	$\sigma_{\max} = \sigma_A = K_t \sigma_{\text{nom}}, \qquad \sigma_{\text{nom}} = \sigma/(1 - d/L)$ $K_t = 2.000 - 1.597\left(\frac{d}{L}\right) + 0.934\left(\frac{d}{L}\right)^2 - 0.337\left(\frac{d}{L}\right)^3$ for $0 \leq d/L \leq 1$

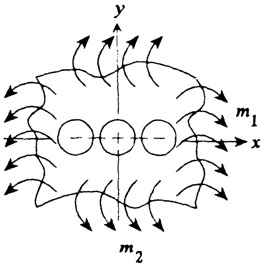
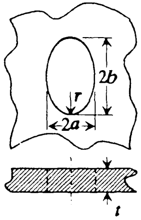
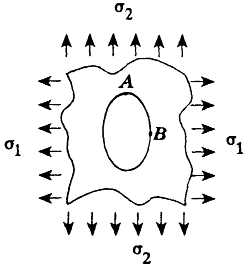
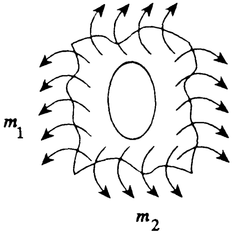
	d. Transverse bending ($\nu = 0.3$)  	Bending about y axis: $\sigma_{\max} = K_t \sigma_{\text{nom}}, \qquad \sigma_{\text{nom}} = 6m/t^2$ for $0 \leq d/L \leq 1$ (1) Simple bending ($m_1 = m,\ m_2 = 0$) $K_t = 1.787 - 0.060\left(\frac{d}{L}\right) - 0.785\left(\frac{d}{L}\right)^2 + 0.217\left(\frac{d}{L}\right)^3$ (2) Cylindrical bending ($m_1 = m,\ m_2 = \nu m$) $K_t = 1.850 - 0.030\left(\frac{d}{L}\right) - 0.994\left(\frac{d}{L}\right)^2 + 0.389\left(\frac{d}{L}\right)^3$ Bending about x axis: $\sigma_{\max} = K_t \sigma_{\text{nom}}, \qquad \sigma_{\text{nom}} = 6m/t^2(1 - d/L)$ for $0 \leq d/L \leq 1$ (1) Simple bending ($m_1 = m,\ m_2 = 0$) $K_t = 1.788 - 1.729\left(\frac{d}{L}\right) + 1.094\left(\frac{d}{L}\right)^2 - 0.111\left(\frac{d}{L}\right)^3$ (2) Cylindrical bending ($m_1 = m,\ m_2 = \nu m$) $K_t = 1.849 - 1.741\left(\frac{d}{L}\right) + 0.875\left(\frac{d}{L}\right)^2 + 0.081\left(\frac{d}{L}\right)^3$

TABLE 6-1 (continued) STRESS CONCENTRATION FACTORS: Holes

6. Single elliptical hole in infinite plate  $r = \frac{a^2}{b}$	a. In-plane normal stress  	(1) Uniaxial tension ($\sigma_1 = \sigma,\ \sigma_2 = 0$): $\sigma_A = K_t\sigma$ $K_t = 1 + \frac{2a}{b} = 1 + 2\sqrt{\frac{a}{r}}$ for $0 < a/b < 10$ and $\sigma_B = -\sigma$ (2) Biaxial tension: For $-1 \le \sigma_2/\sigma_1 \le 1$ and $0.25 \le a/b \le 4$, $\sigma_A = K_{tA}\sigma_1, \qquad K_{tA} = 1 + \frac{2a}{b} - \frac{\sigma_2}{\sigma_1}$ $\sigma_B = K_{tB}\sigma_1, \qquad K_{tB} = \frac{\sigma_2}{\sigma_1}\left(1 + \frac{2b}{a}\right) - 1$ For $\sigma_1 = \sigma_2$, $K_{tA} = 2a/b, \qquad K_{tB} = 2b/a$
	b. Transverse bending  	$\sigma_{\max} = K_t\sigma, \qquad \sigma = 6m/t^2,\ \nu = 0.3$ for $2a/t > 5$ and $0.2 \le a/b < 5$ (1) Simple bending ($m_1 = m,\ m_2 = 0$) $K_t = 1 + \frac{2(1+\nu)(a/b)}{3+\nu}$ for $2a/t > 5$ (2) Cylindrical bending ($m_1 = m,\ m_2 = \nu m$) $K_t = \frac{(1+\nu)[2(a/b)+3-\nu]}{3+\nu}$ (3) Isotropic bending ($m_1 = m_2 = m$) $K_t = 2$ (constant)

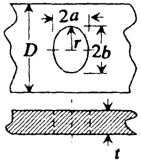
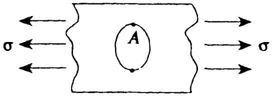
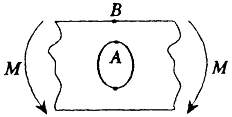
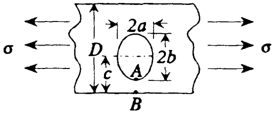
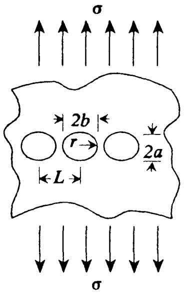
<table>
<tr>
<td rowspan="2">7.
Single elliptical hole in finite-width plate

 </td>
<td>a. Axial tension

 </td>
<td>$\sigma_{\max} = \sigma_A = K_t \sigma_{\text{nom}}, \qquad \sigma_{\text{nom}} = \sigma/(1 - 2a/D)$
$K_t = C_1 + C_2 \frac{2a}{D} + C_3 \left(\frac{2a}{D}\right)^2 + C_4 \left(\frac{2a}{D}\right)^3,$
<table>
<tr><th></th><th>$1.0 \le a/b \le 8.0$</th></tr>
<tr><td>C_1</td><td>$1.109 - 0.188\sqrt{a/b} + 2.086a/b$</td></tr>
<tr><td>C_2</td><td>$-0.486 + 0.213\sqrt{a/b} - 2.588a/b$</td></tr>
<tr><td>C_3</td><td>$3.816 - 5.510\sqrt{a/b} + 4.638a/b$</td></tr>
<tr><td>C_4</td><td>$-2.438 + 5.485\sqrt{a/b} - 4.126a/b$</td></tr>
</table></td>
</tr>
<tr>
<td>b. In-plane bending

 </td>
<td>$\sigma_{\max} = \sigma_A = K_t \sigma_{\text{nom}}, \qquad \sigma_{\text{nom}} = 12Ma/(D^3 - 8a^3)t$
$K_t = C_1 + C_2 \left(\frac{2a}{D}\right) + C_3 \left(\frac{2a}{D}\right)^2$
<table>
<tr><th></th><th>$0.4 \le 2a/D \le 1.0, \qquad 1.0 \le a/b \le 2.0$</th></tr>
<tr><td>C_1</td><td>$1.509 + 0.336(a/b) + 0.155(a/b)^2$</td></tr>
<tr><td>C_2</td><td>$-0.416 + 0.445(a/b) - 0.029(a/b)^2$</td></tr>
<tr><td>C_3</td><td>$0.878 - 0.736(a/b) - 0.142(a/b)^2$</td></tr>
</table>
for $2a/D \le 0.4$, $\sigma_{\max} = \sigma_B = 6M/D^2 t$</td>
</tr>
</table>

TABLE 6-1 (continued) STRESS CONCENTRATION FACTORS: Holes

8. Eccentric elliptical hole in finite-width plate  	Axial tension	Stress on section AB is $\sigma_{\text{nom}} = \frac{\sqrt{1-a/c}}{1-a/c}\frac{1-c/D}{1-(c/D)\left[2-\sqrt{1-(a/c)^2}\right]}$ and $\sigma_{\max} = K_t\sigma_{\text{nom}}$ $K_t = C_1 + C_2\frac{a}{c} + C_3\left(\frac{a}{c}\right)^2 + C_4\left(\frac{a}{c}\right)^3$ for $1.0 \le a/b \le 8.0$ and $0 \le a/c \le 1$ Expressions for C_1, C_2, C_3, and C_4 from case 7a can be used.
9. Infinite row of elliptical holes in infinite-width plate  	Uniaxial tension	$\sigma_{\max} = K_t\sigma_{\text{nom}}, \qquad \sigma_{\text{nom}} = \sigma/(1-2a/L)$ For $0 \le 2a/L \le 0.7$ and $1 \le a/b \le 10$, $K_t = \left[1.002 - 1.016\left(\frac{2a}{L}\right) + 0.253\left(\frac{2a}{L}\right)^2\right]\left(1 + \frac{2a}{b}\right)$

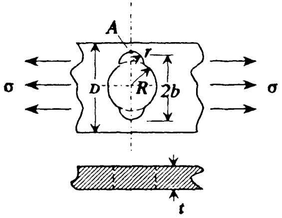
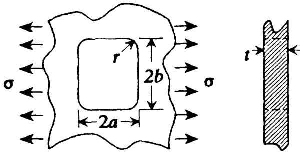
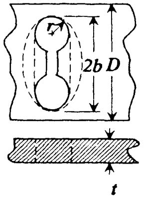
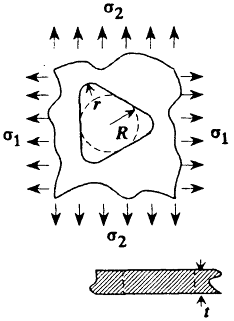
10. Circular hole with opposite semicircular lobes in finite-width plate  	Axial tension	$\sigma_{\max} = K_t \sigma_{\text{nom}}, \qquad \sigma_{\text{nom}} = \sigma/(1-2b/D)$ For $0 \le 2b/D \le 1$, $K_t = K_{t0}\left[1 - \frac{2b}{D} + \left(\frac{6}{K_{t0}} - 1\right)\left(\frac{b}{D}\right)^2 + \left(1 - \frac{4}{K_{t0}}\right)\left(\frac{b}{D}\right)^3\right]$ where for $0.2 < r/R \le 4.0$, $K_{t0} = \frac{\sigma_{\max}}{\sigma} = 2.2889 + \frac{1.6355}{\sqrt{r/R}} - \frac{0.0157}{r/R}$ For infinitely wide plate, $K_t = K_{t0}$.
11. Rectangular hole with rounded corners in infinite-width plate  	Uniaxial tension	$\sigma_{\max} = K_t \sigma$ $K_t = C_1 + C_2 \frac{a}{b} + C_3\left(\frac{a}{b}\right)^2 + C_4\left(\frac{a}{b}\right)^3$ $\begin{array}{c\|c} & 0.05 \le r/2a \le 0.5, \qquad 0.2 \le a/b \le 1.0 \\ \hline C_1 & 14.815 - 22.308\sqrt{r/2a} + 16.298(r/2a) \\ C_2 & -11.201 - 13.789\sqrt{r/2a} + 19.200(r/2a) \\ C_3 & 0.2020 + 54.620\sqrt{r/2a} - 54.748(r/2a) \\ C_4 & 3.232 - 32.530\sqrt{r/2a} + 30.964(r/2a) \end{array}$

TABLE 6-1 (continued) STRESS CONCENTRATION FACTORS: Holes

12. Slot having semicircular ends 	a. Axial tension $a_{\text{eq}} = \sqrt{rb}$ where a_{eq} is width of equivalent ellipse	If the openings such as two holes connected by a slit or an ovaloid are enveloped by an ellipse with the same $2b$ and r, K_t can be approximated by using an equivalent ellipse having the same dimensions $2b$ and r. See cases 6a and 8.
	b. In-plane bending $a_{\text{eq}} = \sqrt{rb}$	Use an equivalent ellipse. See case 6b.
13. Equilateral triangular hole with round corners in infinite-width plate 	a. Uniaxial tension $(\sigma_1 = \sigma,\ \sigma_2 = 0)$	$\sigma_{\max} = K_t \sigma$ For $0.25 \le r/R \le 0.75$ $K_t = 6.191 - 7.215(r/R) + 5.492(r/R)^2$
	b. Biaxial tension $(\sigma_1 = \sigma,\ \sigma_2 = \sigma/2)$	$\sigma_{\max} = K_t \sigma$ For $0.25 \le r/R \le 0.75$ $K_t = 6.364 - 8.885(r/R) + 6.494(r/R)^2$
	c. Biaxial tension $(\sigma_1 = \sigma_2 = \sigma)$	$\sigma_{\max} = K_t \sigma$ For $0.25 \le r/R \le 0.75$ $K_t = 7.067 - 11.099(r/R) + 7.394(r/R)^2$

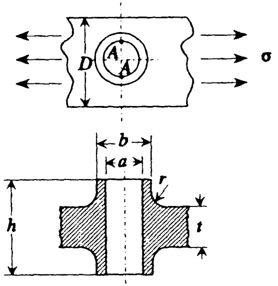
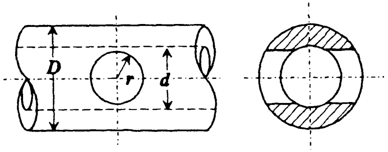
<table>
<tr>
<td rowspan="2">14.
Single symmetrically reinforced circular hole in finite-width plate in tension
 </td>
<td>a. Without fillet ($r = 0$)</td>
<td>$\sigma_{\max} = \sigma_A = K_t\sigma$
where $\sigma_{\max} =$ maximum mean stress for thickness sliced off to plate thickness t. For $b/t = 5.0$,
$$K_t = C_1 + C_2\left(\frac{1}{h/t}\right) + C_3\left(\frac{1}{h/t}\right)^2$$
<table>
<tr><th></th><th>$D/b \geq 4.0,\quad 1 \leq h/t \leq 5\quad$ and $\quad 0.3 \leq a/b \leq 0$</th></tr>
<tr><td>C_1</td><td>$1.869 + 1.196(a/b) - 0.393(a/b)^2$</td></tr>
<tr><td>C_2</td><td>$-3.042 + 6.476(a/b) - 4.871(a/b)^2$</td></tr>
<tr><td>C_3</td><td>$4.036 - 7.229(a/b) + 5.180(a/b)^2$</td></tr>
</table>
</td>
</tr>
<tr>
<td>b. With fillet ($r \neq 0$)</td>
<td>For $r/t \geq 0.6,\ 0.3 \leq a/b \leq 0.7$, and $h/t \geq 3.0$,
$K_t = 3.000 - 2.206\sqrt{R} + 0.948R - 0.142R\sqrt{R}$
$$\text{where } R = \frac{\text{cross-sectional area of added reinforcement}}{\text{cross-sectional area of hole (without added reinforcement)}}$$
$R = \left(\frac{b}{a} - 1\right)\left(\frac{h}{t} - 1\right) + (4 - \pi)\frac{r^2}{at}$</td>
</tr>
</table>

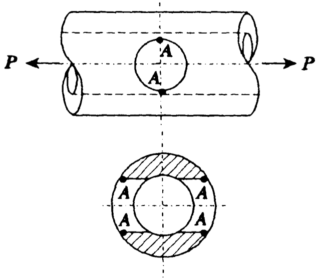
TABLE 6-1 (continued) STRESS CONCENTRATION FACTORS: Holes

15.

Transverse circular hole in round bar or tube



a. Axial tension



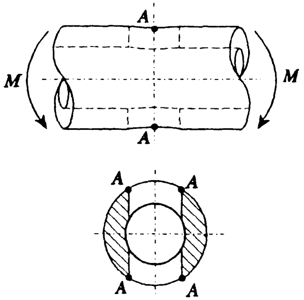
$$\sigma_{\max} = \sigma_A = K_t \sigma_{\text{nom}}$$

where $\sigma_{\text{nom}} = \frac{4P}{\pi(D^2-d^2)}$

$$K_t = C_1 + C_2 \frac{2r}{D} + C_3 \left(\frac{2r}{D}\right)^2$$

	$d/D \le 0.9$, $2r/D \le 0.45$
C_1	3.000
C_2	$0.427 - 6.770(d/D) + 22.698(d/D)^2 - 16.670(d/D)^3$
C_3	$11.357 + 15.665(d/D) - 60.929(d/D)^2 + 41.501(d/D)^3$

b. Bending

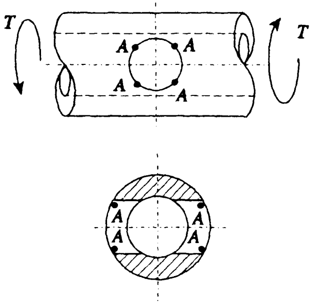


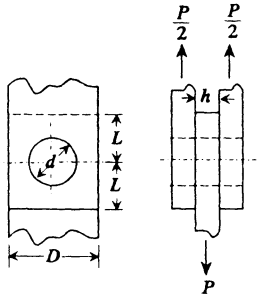
$$\sigma_{\max} = \sigma_A = K_t \sigma_{\text{nom}}$$

where $\sigma_{\text{nom}} = \frac{32MD}{\pi(D^4-d^4)}$

$$K_t = C_1 + C_2 \frac{2r}{D} + C_3 \left(\frac{2r}{D}\right)^2 + C_4 \left(\frac{2r}{D}\right)^3$$

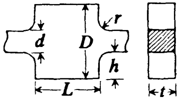
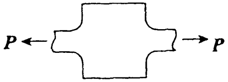
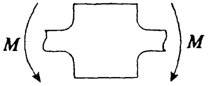
	$d/D \le 0.9$, $2r/D \le 0.4$
C_1	3.000
C_2	$-6.250 - 0.585(d/D) + 3.115(d/D)^2$
C_3	$41.000 - 1.071(d/D) - 6.746(d/D)^2$
C_4	$-45.000 + 1.389(d/D) + 13.889(d/D)^2$

<table>
<tr>
<td></td>
<td>c. Torsion


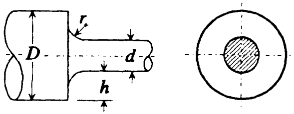
</td>
<td>
$\sigma_{\max} = \sigma_A = K_t \tau_{\text{nom}}, \qquad \tau_{\text{nom}} = 16TD/\pi(D^4 - d^4)$
$K_t = C_1 + C_2 \frac{2r}{D} + C_3 \left(\frac{2r}{D}\right)^2 + C_4 \left(\frac{2r}{D}\right)^3$
<table>
<tr><th></th><th>$2r/d \le 0.4, \qquad d/D \le 0.8$</th></tr>
<tr><td>C_1</td><td>4.000</td></tr>
<tr><td>C_2</td><td>$-6.055 + 3.184(d/D) - 3.461(d/D)^2$</td></tr>
<tr><td>C_3</td><td>$32.764 - 30.121(d/D) + 39.887(d/D)^2$</td></tr>
<tr><td>C_4</td><td>$-38.330 + 51.542\sqrt{d/D} - 27.483(d/D)$</td></tr>
</table>
Maximum stress occurs inside hole on hole surface, near outer surface of bar
Maximum shear stress concentration factor
$K_{tS} = \tau_{\max}/\tau_{\text{nom}} = \frac{1}{2}K_t$
</td>
</tr>
<tr>
<td>**16.**
Round pin joint with closely fitting pin in finite-width plate</td>
<td>Tension


</td>
<td>
Nominal stress based on net section:
$\sigma_{\max} = K_{ta}\sigma_{na}, \quad \sigma_{na} = P/(D - d)h$
Nominal stress based on bearing area:
$\sigma_{\max} = K_{tb}\sigma_{nb}, \quad \sigma_{nb} = P/dh$
For $0.15 \le d/D \le 0.75,\ L/D \ge 1.0,$
$K_{ta} = 12.882 - 52.714\left(\frac{d}{D}\right) + 89.762\left(\frac{d}{D}\right)^2 - 51.667\left(\frac{d}{D}\right)^3$
$K_{tb} = 0.2880 + 8.820\left(\frac{d}{D}\right) - 23.196\left(\frac{d}{D}\right)^2 + 29.167\left(\frac{d}{D}\right)^3$
</td>
</tr>
</table>

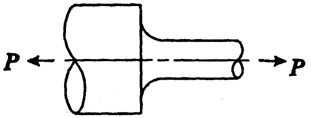
TABLE 6-1 (continued) STRESS CONCENTRATION FACTORS: Fillets

<table>
<tr><th colspan="3">III. Fillets</th></tr>
<tr><th>Type of Stress Raiser</th><th>Loading Conditions</th><th>Stress Concentration Factor</th></tr>
<tr>
<td rowspan="2">1.
Opposite shoulder fillets in stepped flat bar
 </td>
<td>a. Axial tension
 </td>
<td>$\sigma_{\max} = K_t \sigma_{\text{nom}}, \qquad \sigma_{\text{nom}} = P/td$
$K_t = C_1 + C_2 \frac{2h}{D} + C_3 \left(\frac{2h}{D}\right)^2 + C_4 \left(\frac{2h}{D}\right)^3$
where $\frac{L}{D} > -1.89\left(\frac{r}{d} - 0.15\right) + 5.5$
<table>
<tr><th></th><th>$0.1 \le h/r \le 2.0$</th><th>$2.0 \le h/r \le 20.0$</th></tr>
<tr><td>C_1</td><td>$1.006 + 1.008\sqrt{h/r} - 0.044h/r$</td><td>$1.020 + 1.009\sqrt{h/r} - 0.048h/r$</td></tr>
<tr><td>C_2</td><td>$-0.115 - 0.584\sqrt{h/r} + 0.315h/r$</td><td>$-0.065 - 0.165\sqrt{h/r} - 0.007h/r$</td></tr>
<tr><td>C_3</td><td>$0.245 - 1.006\sqrt{h/r} - 0.257h/r$</td><td>$-3.459 + 1.266\sqrt{h/r} - 0.016h/r$</td></tr>
<tr><td>C_4</td><td>$-0.135 + 0.582\sqrt{h/r} - 0.017h/r$</td><td>$3.505 - 2.109\sqrt{h/r} + 0.069h/r$</td></tr>
</table>
For cases where $L/D < -1.89(r/d - 0.15) + 5.5$, see Ref. [6.1].</td>
</tr>
<tr>
<td>b. In-plane bending
 </td>
<td>$\sigma_{\max} = K_t \sigma_{\text{nom}}, \qquad \sigma_{\text{nom}} = 6M/td^2$
$K_t = C_1 + C_2 \frac{2h}{D} + C_3 \left(\frac{2h}{D}\right)^2 + C_4 \left(\frac{2h}{D}\right)^3$
where $\frac{L}{D} > -2.05\left(\frac{r}{d} - 0.025\right) + 2.0$
<table>
<tr><th></th><th>$0.1 \le h/r \le 2.0$</th><th>$2.0 \le h/r \le 20.0$</th></tr>
<tr><td>C_1</td><td>$1.006 + 0.967\sqrt{h/r} + 0.013h/r$</td><td>$1.058 + 1.002\sqrt{h/r} - 0.038h/r$</td></tr>
<tr><td>C_2</td><td>$-0.270 - 2.372\sqrt{h/r} + 0.708h/r$</td><td>$-3.652 + 1.639\sqrt{h/r} - 0.436h/r$</td></tr>
<tr><td>C_3</td><td>$0.662 + 1.157\sqrt{h/r} - 0.908h/r$</td><td>$6.170 - 5.687\sqrt{h/r} + 1.175h/r$</td></tr>
<tr><td>C_4</td><td>$-0.405 + 0.249\sqrt{h/r} - 0.200h/r$</td><td>$-2.558 + 3.046\sqrt{h/r} - 0.701h/r$</td></tr>
</table></td>
</tr>
</table>

2.
Shoulder fillet in stepped circular shaft



a. Axial tension

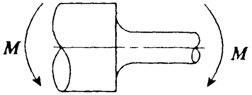


$\sigma_{\max} = K_t \sigma_{\text{nom}}, \qquad \sigma_{\text{nom}} = 4P/\pi d^2$

$$K_t = C_1 + C_2 \frac{2h}{D} + C_3 \left(\frac{2h}{D}\right)^2 + C_4 \left(\frac{2h}{D}\right)^3$$

	$0.1 \le h/r \le 2.0$	$2.0 \le h/r \le 20.0$
C_1	$0.926 + 1.157\sqrt{h/r} - 0.099h/r$	$1.200 + 0.860\sqrt{h/r} - 0.022h/r$
C_2	$0.012 - 3.036\sqrt{h/r} + 0.961h/r$	$-1.805 - 0.346\sqrt{h/r} - 0.038h/r$
C_3	$-0.302 + 3.977\sqrt{h/r} - 1.744h/r$	$2.198 - 0.486\sqrt{h/r} + 0.165h/r$
C_4	$0.365 - 2.098\sqrt{h/r} + 0.878h/r$	$-0.593 - 0.028\sqrt{h/r} - 0.106h/r$

b. Bending



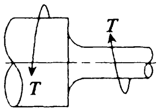
$\sigma_{\max} = K_t \sigma_{\text{nom}}, \qquad \sigma_{\text{nom}} = 32M/\pi d^3$

$$K_t = C_1 + C_2 \frac{2h}{D} + C_3 \left(\frac{2h}{D}\right)^2 + C_4 \left(\frac{2h}{D}\right)^3$$

	$0.1 \le h/r \le 2.0$	$2.0 \le h/r \le 20.0$
C_1	$0.947 + 1.206\sqrt{h/r} - 0.131h/r$	$1.232 + 0.832\sqrt{h/r} - 0.008h/r$
C_2	$0.022 - 3.405\sqrt{h/r} + 0.915h/r$	$-3.813 + 0.968\sqrt{h/r} - 0.260h/r$
C_3	$0.869 + 1.777\sqrt{h/r} - 0.555h/r$	$7.423 - 4.868\sqrt{h/r} + 0.869h/r$
C_4	$-0.810 + 0.422\sqrt{h/r} - 0.260h/r$	$-3.839 + 3.070\sqrt{h/r} - 0.600h/r$

TABLE 6-1 (continued) STRESS CONCENTRATION FACTORS: Fillets

c. Torsion



$$\tau_{\max} = K_t \tau_{\text{nom}}, \qquad \tau_{\text{nom}} = 16T/\pi d^3$$

$$K_t = C_1 + C_2 \frac{2h}{D} + C_3 \left(\frac{2h}{D}\right)^2 + C_4 \left(\frac{2h}{D}\right)^3$$

	$0.25 \le h/r \le 4.0$
C_1	$0.905 + 0.783\sqrt{h/r} - 0.075h/r$
C_2	$-0.437 - 1.969\sqrt{h/r} + 0.553h/r$
C_3	$1.557 + 1.073\sqrt{h/r} - 0.578h/r$
C_4	$-1.061 + 0.171\sqrt{h/r} + 0.086h/r$

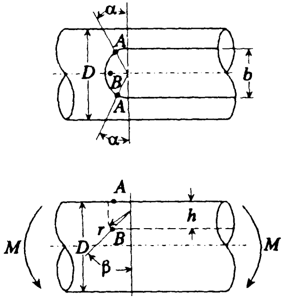
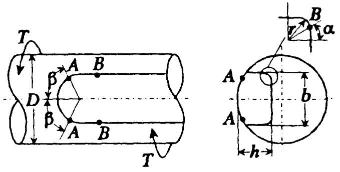
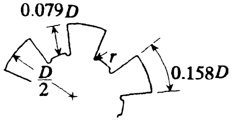
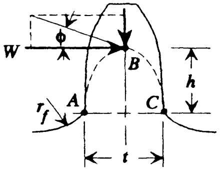
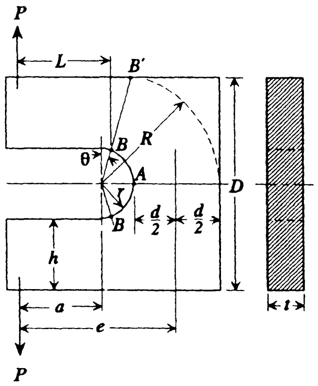
IV. Miscellaneous Elements		
Type of Stress Raiser	Loading Conditions	Stress Concentration Factor
1. Round shaft with semicircular end key seat	a. Bending 	$\sigma_{\max} = K_t\sigma, \qquad \sigma = 32M/\pi D^3$ $b = \frac{1}{4}D, \qquad h = \frac{1}{8}D, \qquad \alpha = 10°, \qquad \beta = 15°$ (1) At location A on surface: $K_{tA} = 1.6$ (2) At location B at end of keyway: $K_{tB} = 1.426 + 0.1643\left(\frac{0.1}{r/D}\right) - 0.0019\left(\frac{0.1}{r/D}\right)^2$ where $0.005 \le r/D \le 0.04$ $D \le 6.5$ in. $h/D = 0.125$ For $D > 6.5$ in., it is suggested that the K_{tB} values for $r/D = 0.0208$ be used.
	b. Torsion 	$h = \frac{1}{8}D, \qquad b = D/r, \qquad \alpha = 15°, \qquad \beta = 50°$ (1) At location A on surface: $K_{tA} = \sigma_{\max}/\tau \simeq 3.4, \qquad \tau = 16T/\pi D^3$ (2) At location B in fillet: $K_{tB} = \sigma_{\max}/\tau$ $= 1.953 + 0.1434\left(\frac{0.1}{r/D}\right) - 0.0021\left(\frac{0.1}{r/D}\right)^2$ for $0.005 \le r/D \le 0.07$

TABLE 6-1 (continued) STRESS CONCENTRATION FACTORS: Miscellaneous Elements

2. Splined shaft	a. Torsion 	For an eight-tooth spline $K_{tS} = \tau_{\max}/\tau, \qquad \tau = 16T/\pi D^3$ For $0.01 \le r/D \le 0.04$ $K_{tS} = 6.083 - 14.775\left(\frac{10r}{D}\right) + 18.250\left(\frac{10r}{D}\right)^2$
3. Gear teeth 	Bending plus some compression A and C are points of tangency of inscribed parabola ABC with tooth profile b = tooth width normal to plane of figure r_f = minimum radius of tooth fillet W = load per unit length of tooth face ϕ = angle between load W and normal to tooth face	Maximum stress occurs at fillet on tension side at base of tooth $\sigma_{\max} = K_t \sigma_{\text{nom}}, \qquad \sigma_{\text{nom}} = \frac{6Wh}{bt^2} - \frac{W}{bt}\tan\phi$ For 14.5° pressure angle, $K_t = 0.22 + \left(\frac{t}{r_f}\right)^{0.2}\left(\frac{t}{h}\right)^{0.4}$ For 20° pressure angle, $K_t = 0.18 + \left(\frac{t}{r_f}\right)^{0.15}\left(\frac{t}{h}\right)^{0.45}$

4.
U-shaped member



where $\theta = 20°$
$e = L + r + d/2$

For position A,

$$K_{tA} = \frac{\sigma_{\max} - P/td}{6Pe/td^2}$$

For position B,

$$K_{tB} = \frac{\sigma_{\max}}{PLc_B/I_B}$$

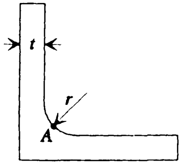
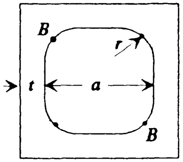
where I_B/c_B = section modulus at section in question (section BB')

(1) For square outer corners

$\frac{e}{r} = \frac{e}{h} = \frac{e}{d}$ $1.5 \leq \frac{e}{r} \leq 4.5$	$K_{tA} = 0.194 + 1.267\left(\frac{e}{r}\right) - 0.455\left(\frac{e}{r}\right)^2 + 0.050\left(\frac{e}{r}\right)^3$ $K_{tB} = 4.141 - 2.760\left(\frac{e}{r}\right) + 0.838\left(\frac{e}{r}\right)^2 - 0.082\left(\frac{e}{r}\right)^3$
$\frac{e}{2r} = \frac{e}{2h} = \frac{e}{d}$ $1.0 \leq \frac{e}{2r} \leq 2.5$	$K_{tA} = 0.800 + 1.147\left(\frac{e}{2r}\right) - 0.580\left(\frac{e}{2r}\right)^2 + 0.093\left(\frac{e}{2r}\right)^3$ $K_{tB} = 7.890 - 11.107\left(\frac{e}{2r}\right) + 6.020\left(\frac{e}{2r}\right)^2 - 1.053\left(\frac{e}{2r}\right)^3$

TABLE 6-1 (continued) STRESS CONCENTRATION FACTORS: Miscellaneous Elements

	$\frac{d}{r} = \frac{d}{h}$ $0.75 \le \frac{d}{r} \le 2.0$	When $a = 3r$, $K_{tA} = 1.143 + 0.074\left(\frac{d}{r}\right) + 0.026\left(\frac{d}{r}\right)^3$ $K_{tB} = 1.276$
		When $a = r$, $K_{tA} = 0.714 + 1.237\left(\frac{d}{r}\right) - 0.891\left(\frac{d}{r}\right)^2 + 0.239\left(\frac{d}{r}\right)^3$ $K_{tB} = 1.374$
	$\frac{d}{r} = \frac{h}{r}$ $1.0 \le \frac{d}{r} \le 7.0$	For $a = 3r$, $K_{tA} = 0.982 + 0.303\left(\frac{d}{r}\right) - 0.017\left(\frac{d}{r}\right)^2$ $K_{tB} = 1.020 + 0.235\left(\frac{d}{r}\right) - 0.015\left(\frac{d}{r}\right)^2$
		For $a = r$, $K_{tA} = 1.010 + 0.281\left(\frac{d}{r}\right) - 0.012\left(\frac{d}{r}\right)^2$ $K_{tB} = 0.200 + 1.374\left(\frac{d}{r}\right) - 0.412\left(\frac{d}{r}\right)^2 + 0.037\left(\frac{d}{r}\right)^3$

	(2) For rounded outer corners	
	$\frac{R}{r} = \frac{R}{d} = \frac{R}{h}$ $2.0 \le \frac{R}{r} \le 2.75$	For $a = 3r$, $K_{tA} = 48.959 - 60.004 \left(\frac{R}{r}\right) + 24.933 \left(\frac{R}{r}\right)^2 - 3.427 \left(\frac{R}{r}\right)^3$ $K_{tB} = 79.769 - 98.346 \left(\frac{R}{r}\right) + 40.806 \left(\frac{R}{r}\right)^2 - 5.610 \left(\frac{R}{r}\right)^3$
		For $a = r$, $K_{tA} = 27.714 - 31.859 \left(\frac{R}{r}\right) + 12.625 \left(\frac{R}{r}\right)^2 - 1.648 \left(\frac{R}{r}\right)^3$ $K_{tB} = 81.344 - 99.133 \left(\frac{R}{r}\right) + 40.740 \left(\frac{R}{r}\right)^2 - 5.560 \left(\frac{R}{r}\right)^3$
5. Angles and box sections  	Torsion	(1) For angle section: $\tau_{\max} = \tau_A = K_t \tau$ $K_t = 6.554 - 16.077\sqrt{\frac{r}{t}} + 16.987 \left(\frac{r}{t}\right) - 5.886\sqrt{\frac{r}{t}} \left(\frac{r}{t}\right)$ where $0.1 \le r/t \le 1.4$ (2) For box section: $\tau_{\max} = \tau_B = K_t \tau$ $K_t = 3.962 - 7.359 \left(\frac{r}{t}\right) + 6.801 \left(\frac{r}{t}\right)^2 - 2.153 \left(\frac{r}{t}\right)^3$ where a is 15–20 times larger than t, $0.2 \le r/t \le 1.4$

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[a] Much of this material is based on Ref. [6.1].